Estimating trade network models without network econometrics*

Peter H. Egger[†]
ETH Zurich
CEPR, CESifo, GEP, WIFO

Kevin E. Staub[‡]
University of Melbourne
IZA

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Abstract

This paper studies structural gravity network models à la Behrens, Ertur and Koch (2012). Such models provide an elegant linearization of nonlinear structural gravity models of international trade, and have a wide range of other applications to bilateral flow data such as investments and migration. In the context of trade, these models have the desirable feature that they account for so-called multilateral resistance (or aggregate price index) terms in a theory-consistent way. Earlier research had proposed applying network-econometric techniques for estimating such models. We exploit the structure of the model to propose simple alternative estimators that do not require any specific network methods, making the structural network model amenable to broader use by practitioners. We show that all structural model parameters can be recovered from a linear OLS regression that uses a properly-weighted average of the dependent variable in a control function. Our control-function approach can also be implemented with simple nonlinear estimators instead of OLS, such as Poisson pseudo maximum likelihood or other generalized linear model estimators.

Keywords: Gravity models; Multilateral resistance terms; Network models.

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†ETH Zurich, Department of Management, Technology, and Economics, Weinbergstr. 35, 8092 Zurich, Switzerland; E-mail: egger@kof.ethz.ch; + 41 44 632 41 08.

‡Department of Economics, 111 Barry Street, The University of Melbourne, 3010 VIC, Australia; E-mail: kevin.staub@unimelb.edu.au; +61 3 903 53776.

1 Introduction

The gravity equation—describing aggregate demand for goods or services between any pair of countries—is among the most successful concepts in all of economics (see Leamer and Levinsohn, 1995). Its popularity derives from the fact that it nests a wide variety of isomorphic structural models of aggregate bilateral demand such as endowment-economy, Ricardian, and monopolistic-competition increasing-returns-to-scale models with a fixed markup (see Eaton and Kortum, 2002; Anderson and van Wincoop, 2003; Arkolakis, Costinot and Rodríguez-Clare, 2012; Bergstrand, Egger and Larch, 2013; Baltagi, Egger and Pfaffermayr, 2015).

The main reason why gravity equations are rarely estimated in their structural form is that they are nonlinear in parameters after taking logarithms. Rather, practitioners use either country fixed effects, which may be inefficient, or theory-based linear approximations in estimation. Behrens, Ertur and Koch (2012, henceforth BEK) introduced such a linearization in a version of a constant-elasticity-of-substitution model, where price indices can be expressed as implicit functions of trade flows. The linearized equilibrium system leads to an econometric specification in which trade flows between two countries depend on trade flows between all trading partners, thus exhibiting the characteristics of a network-weighted model. To deal with this interdependence among observations, BEK adapt methods from the literature on network econometrics that account for simultaneous, cross-sectionally autoregressive structures. While the model has since been used, an existing barrier to its further dissemination lies in the computationally demanding network-based estimation.

In this paper, we propose estimators that exploit the structural form of the same network model to recover the model's structural parameters. In contrast to the previous network-based approach to estimating the model, the proposed estimators are simple OLS (or Poisson PML—pseudo maximum likelihood) estimators. In our preferred proposed approach, the network structure is fully captured through the inclusion of an appropriate regressor that serves in a control function to address the endogeneity resulting from the model's network structure. As a result, the proposed estimator is fully efficient in the sense of there not being any efficiency loss due to estimating the linear approximation rather than the true non-linearized model. Constructing this control function is straightforward as it is just a weighted average of the data and does neither require instruments nor additional estimation steps. Therefore, the proposed alternative estimators are easily implementable in practice, providing a low-cost way for practitioners to estimate this popular structural gravity network model, either applied to international trade flows as in the original BEK paper or applied to other bilateral flow data such as immigration or financial investment.

We examine the properties of the structural gravity model and this linearization as well as the finite sample performance of the estimators in a series of numerical Monte Carlo experiments. The simulation data are obtained by randomly drawing exogenous variables from real-world data and then solving for endogenous variables according to the theory-based structural general equilibrium model. As we show theoretically and confirm in the simulations, the only source of bias in a proposed reduced-form OLS estimation stems from the original linearization of the model. Overall, the simulation results demonstrate that this bias is moderate and shrinks towards zero with increasing sample size. Moreover, the preferred new control-function OLS approach does not suffer from this linearization bias and is unbiased in all sample sizes.

We continue with presenting the generic structural gravity model and its linearization following BEK in Section 2. we show how this model can be estimated by linear (or generalized linear) methods in Section 3. A numerical assessment of the estimators is provided in Section 4, followed by our conclusions in Section 5.

2 A linearized structural gravity network model of trade

We consider a standard gravity equation derived from utility maximization subject to income constraints, which can be represented for exporter i and importer j as

$$Z_{ij} \equiv \frac{X_{ij}}{Y_i Y_j} = \frac{C_i^{\alpha-1} T_{ij}^{\alpha}}{\sum_{k=1}^{N} L_k C_k^{\alpha} T_{kj}^{\alpha}} \qquad C_i = L_i^{-1} \sum_{j=1}^{N} X_{ij} = \sum_{j=1}^{N} \frac{C_i^{\alpha} T_{ij}^{\alpha} C_j L_j}{\sum_{k=1}^{N} L_k C_k^{\alpha} T_{kj}^{\alpha}}. \tag{1}$$

Here, Z_{ij} are aggregate bilateral exports, X_{ij}, normalized by exporter and importer GDP, Y_i and Y_j, respectively. The variable C_i represents the costs per unit of a single or a bundle of factors L_i, T_{ij} are bilateral trade costs, and L_i is size (up to scale) of a market measured in terms of its factor supply (see Arkolakis et al., 2012). As mentioned, (1) is compatible with a variety of aggregate bilateral demand models such as endowment-economy, Ricardian, and monopolistic-competition-increasing returns-to-scale models (see the appendix for more details).

Equation (1) shows that the log of Z_{ij}, z_{ij}, is a log-nonlinear function of $\{C_i; T_{ij}; L_i\}$. The key structural parameter $\alpha = (-\infty, 0)$ reflects the partial response of trade with respect to changes in trade costs (see Dixit and Stiglitz, 1977, or Eaton and Kortum, 2002). Through (1), upon choice of a numéraire cost ($C_1 = 1$) and for a given α, the $N-1$ endogenous values of C_i are determined by $N-1$ equations for given (exogenous) values of L_i and T_{ij}^{α}.[1]

BEK take the logarithm of equation (1) and derive a first-order approximation around the point $\alpha = 0$. To provide a compact notation, let us generally use the convention that lower-case letters refer to variables in logarithms. and let us refer to N-size vectors and $N \times N$ square matrices by subscripts N and NN, respectively.[2] Letting the world endowment size be denoted by $L \equiv \sum_{k=1}^{N} L_k$, define the following vectors and matrices:

[1]In a Dixit-Stiglitz-Krugman-type model the size of countries is parameterized by the endowment L_i. In an Eaton-Kortum Ricardian economy this would be productivity, and in an Armington economy it would be a preference shifter. For the purpose of the arguments in this paper, these differences are only semantic.

[2]Recall that N parameterizes the number of countries, and N^2 the number of country pairs, including every domestic relation where $i = j$ as one such pair for every country i.

(i) the $N \times 1$ column vector ω_N, whose ith row element is $L_i/L \in (0,1)$,

(ii) the $N \times 1$ column vector of ones ι_N,

(iii) the $N \times N$ asymmetric matrix $W_{NN} = \iota_N \omega_N'$ which in every row of column j contains the same elements L_j/L in all the rows,

(iv) the identity matrix $I_{NN} = diag(\iota_N)$.

Let us stack all observations on Z_{ij} across exporters i for a given importer j into the $N \times 1$ vector of log bilateral normalized exports $z_{jN} = (z_{ij})$, stack log world endowment into an $N \times 1$ vector $l_N = (l) = l\iota_N$ with identical row entries, stack log unit factor-bundle costs into the $N \times 1$ vector $c_N = (c_i)$, and stack log ad-valorem trade costs across all exporters i for a given importer j into the $N \times 1$ vector $t_{jN} = (t_{ij})$. Using this notion, BEK arrive at a log-transformed and linearized counterpart to (1):

$$z_{jN} = \alpha W_{NN} z_{jN} + (\alpha - 1)(l_N + c_N) + \alpha(I_{NN} - W_{NN})t_{jN} + \eta_{jN}, \tag{2}$$

where η_{jN} is the approximation error due to linearization, which only varies across importers j but not exporters i.[3,4] In its role as the coefficient on $W_{NN}z_{jN}$, BEK refer to α in equation (2) as the *autoregressive interaction coefficient*. They do so, because $W_{NN}z_{jN}$ captures the interdependence of bilateral exporters across countries and provides an intuitive network measure of "spatial interaction" or "competition". It is customary in empirical work to further specify $t_{ij} = \sum_{h=1}^{H} \gamma_h d_{h,ij}$ (see Anderson and van Wincoop, 2003, 2004, Eaton and Kortum, 2002, and many others), where $d_{h,ij}$ are H-many observable trade-cost variables in logs such as bilateral log bilateral distance. What is then estimated on $d_{h,ij}$ are the compound parameters $\alpha\gamma_h$.

The reduced form that directly corresponds to (2) is

$$z_{jN} = (I_{NN} - \alpha W_{NN})^{-1}[(\alpha - 1)(l_N + c_N) + \alpha(I_{NN} - W_{NN})t_{jN} + \eta_{jN}]. \tag{3}$$

Existence and uniqueness of the latter requires two things: (i) that $(I_{NN} - \alpha W_{NN})$ is invertible, and (ii) that $[(\alpha - 1)(l_N + c_N) + \alpha(I_{NN} - W_{NN})t_{jN} + \eta_{jN}]$ is an $N \times 1$ vector with finite elements. The latter property is trivially fulfilled in this setting. In contrast, the invertibility of the matrix $(I_{NN} - \alpha W_{NN})$ is not obvious. The reason is that the weighting matrix W_{NN} has nonzero diagonal elements. Hence, the network features self-loops (see Newman, 2018), which induce what Manski

[3] This is equation (11) in BEK. The notation in BEK uses the parametrization $\alpha \equiv 1 - \sigma$ and stacks further across all j. BEK linearize the model about an importer-specific term, the log ideal consumer-price term. As this term lacks variation across exporters i, the corresponding approximation error, too, is importer-specific.

[4] After defining $Y \equiv \sum_{i=1}^{N} Y_i$, at the approximation point $\alpha = 0$ of the model, we obtain $X_{ij} = \frac{L_i Y_j}{L}$ and $Y_i = \sum_{j=1}^{N} X_{ij} = \frac{L_i Y}{L}$, which implies factor-cost equalization, $C_i = C$. Choosing C as the numéraire, we obtain $X_{ij} = \frac{L_i L_j}{L}$. Then, trade costs are irrelevant, and the variance of log bilateral exports, $x_{ij} = \ln X_{ij}$, is fully determined by the variation in exporter- and importer-specific log factor endowments across countries or regions i and j.

(1993) called the "reflection problem" (see also Bramoullé et al., 2009). It was the presence of these self-loops, which led BEK to reformulate equation (3) in a way that complicates estimation tremendously. However, we are able to establish the analytical invertibility of $(I_{NN} - \alpha W_{NN})$ in spite of the presence of self-loops and discuss the respective properties of the inverse below.

3 Novel insights and econometric methods

3.1 Properties of the linearized gravity network model

Despite network matrices with self-loops falling outside the assumptions covered in the aforementioned literature in econometrics and statistics, such network matrices are quite common in economics. The so-called Leontief inverse—which is based on a selling-sector revenue-scaled inter-sector input-output-flow matrix—is one of the most prominent examples. Earlier work established results regarding the existence and uniqueness of such problems involving Leontief-type inverses with self-loops (see Woodbury, 1949, Lampert and Scholtes, 2023, Bellido and Prieto-Martínez, 2024), and we build on these results to show the existence of the reduced form for the linearized gravity network model in the following lemma.

Lemma 1 (Inverse of $(I_{NN} - \alpha W_{NN})$)**.** *Let each country exhibit an endowment that is positive and finite with $0 < \underline{L} \leq L_i \leq \overline{L} < \infty$, where $\{\underline{L}, \overline{L}\}$ are bounding constants for country endowments. Let the parameter α be bounded in the compact interval $-\infty < \underline{\alpha} \leq \alpha \leq \overline{\alpha} < 0$, where $\{\underline{\alpha}, \overline{\alpha}\}$ are bounding constants for α.*

Then, the inverse of the matrix $(I_{NN} - \alpha W_{NN})$ exists and is unique, and it is given by the Sherman-Morrison-Woodbury formula as

$$(I_{NN} - \alpha W_{NN})^{-1} = I_{NN} + \frac{\alpha}{1-\alpha} W_{NN}. \tag{4}$$

Proof. Let us use $a = -\alpha$ and the $M \times 1$ vector $a_N = a\iota_N$. Note that I_{NN} is a special case of an invertible square matrix with real-valued entries. And note that a_N and ω_N are special cases of column vectors with real-valued entries. In general, the Sherman-Morrison-Woodbury formula for a non-singular, real-valued $N \times N$ matrix I_{NN} and real-valued $N \times 1$ column vectors (a_N, ω_N) forming the $N \times N$ matrix $a_N \omega_N' = -\alpha W_{NN}$ states that (see Sherman and Morrison, 1950; Woodbury, 1950; Riedel, 1992; Hao and Simoncini, 2021)

$$(I_{NN} + a_N \omega_N')^{-1} = I_{NN}^{-1} - \frac{I_{NN}^{-1} a_N \omega_N' I_{NN}^{-1}}{1 + \omega_N' a_N}.$$

Clearly, $I_{NN}^{-1} = I_{NN}$ with an identity matrix. Moreover $I_{NN}^{-1} a_N \omega_N' I_{NN}^{-1} = a_N \omega_N' = -\alpha W_{NN}$ and $1 + \omega_N' a_N = 1 + a = 1 - \alpha$, because the row entries of ω_N' are all positive, smaller than unity, and sum up to one. Hence,

$$(I_{NN} - \alpha W_{NN})^{-1} = I_{NN} + \frac{\alpha}{1-\alpha} W_{NN}$$

for every real-valued, non-zero scalar α in the admissible parameter region $[\underline{\alpha}, \overline{\alpha}]$ and any row-normalized $N \times N$ network matrix $W_{NN} = \iota_N \omega'_N$ based on the real-valued weight vector ω'_N, whose elements sum up to unity. □

Our result on the existence of the inverse in Lemma 1 and therefore the existence of the reduced form (3) contributes to a recent literature in international economics which studies the graph stability and finiteness of responses in shocks of graphs in trade and migration models with self-loops (Allen et al., 2020; Kucheryavyy et al., 2023; Allen et al., 2024; Kucheryavyy et al., 2024; Bifulco et al., 2025). A key difference of our model to this literature is that here the model is linear in parameters. Thus, while this literature has to impose (strong) restrictions on parameters to establish the uniqueness of equilibrium responses to shocks (e.g., the exact or near symmetry or the relative magnitude of the elements t_{ij} of the trade-cost matrix), existence in the present case follows under relatively general conditions, as we have shown.[5]

Based on Lemma 1, we can further simplify the reduced form of the nonstochastic model in (3). To this end, note that any invariant vector v_N such as l_N or η_{jN} has the property that $I_{NN} v_N = W_{NN} v_N = v_N$. Moreover, note that W_{NN} is idempotent. To see this, recall that $W_{NN} = (\iota_N \omega'_N)$. Therefore, $W_{NN}^2 = (\iota_N \omega'_N)(\iota_N \omega'_N) = \iota_N(\omega'_N \iota_N)\omega'_N$, where $(\omega'_N \iota_N) = 1$ by design. Using these results, we can state the right-hand-side terms of (3) as

$$\begin{aligned}
(I_{NN} - \alpha W_{NN})^{-1}(\alpha - 1)(l_N + c_N) &= -(l_N - c_N) + \alpha(I_{NN} - W_{NN})c_N, \\
(I_{NN} - \alpha W_{NN})^{-1}\alpha(I_{NN} - W_{NN})t_{jN} &= \alpha(I_{NN} - W_{NN})t_{jN}, \\
(I_{NN} - \alpha W_{NN})^{-1}\eta_{jN} &= \frac{1}{1-\alpha}\eta_{jN},
\end{aligned}$$

so that, after rearranging terms, we can write the reduced form (3) as

$$z_{jN} = -(l_N - c_N) + \alpha(I_{NN} - W_{NN})(c_N + t_{jN}) + \frac{1}{1-\alpha}\eta_{jN}. \tag{5}$$

Further, by pre-multiplying both sides of (5) with W_{NN} we obtain

$$W_{NN} z_{jN} = -W_{NN}(l_N + c_N) + \frac{1}{1-\alpha}\eta_{jN}, \tag{6}$$

[5] It should also be noted that the structure and configuration of the network $W_{NN} = \iota_N \omega'_N$ falls strictly outside the catalog of assumptions invoked in most of the network-econometrics and social-interactions literature (see Kelejian and Prucha, 1999, Lee, 2003, 2004, Bramoullé et al., 2009, and many others). The make-up of the matrix W_{NN} is also special in that it relies on the distribution of node (here, a node being a country) weights in the overall network. The weight of node i in the network is the same for any node j, whereby network-weighted averages of node characteristics across all N nodes are the same for every node i. To the best of our knowledge, the network- and social-interactions literature does not offer insights for this situation. However, we are faced with such a problem for theoretical reasons.

where $-W_{NN}(l_N + c_N)$ is a constant.

3.2 Novel estimators for the linearized gravity network model

The forms of the reduced form equation (5) and of the autoregressive network lag (6) that appears in the structural model (2) have important consequences for the feasibility of econometric approaches to estimate this linearized gravity network model. Specifically, earlier work proposes instrumental-variable estimators to estimate the structural form of autoregressive network models that are linear in parameters as is the model in (2) (see Kelejian and Prucha, 1998, Lee, 2003, or Kelejian et al., 2004). A general result of our paper is that such an approach has limitations for all network models where the weights matrix is constructed as $W_{NN} = \iota_N \omega'_{NN}$ with ω'_N being a weights vector that will appear in each row of the idempotent matrix W_{NN}. The reason is that said instrumental-variables approach relies on instruments generated based on the power series $(I_{NN}, W_{NN}, W^2_{NN}, ...)$ applied to the exogenous regressors other than the (endogenous) network-weighted outcome variable in the model. With an idempotent W_{NN}, this series and associated matrix of elements is reduced to (I_{NN}, W_{NN}). Whenever the structural model includes regressors that appear weighted with W_{NN} on the right-hand side (as is the case with t_{jN} here), these variables are completely lost as instruments. In the present context, t_{jN} is the only regressor which varies across all observations, which precludes instrumental variable estimation of the structural model form in (2). Two-stage least-squares approaches require instruments for $W_{NN} z_{jN}$ which satisfy the exogeneity assumption. Equation (6) shows that there are no suitable instruments for $W_{NN} z_{jN}$ in the model at hand. The nature of $\{W_{NN}, l_N, c_N, \eta_{jN}\}$ and the parameter restrictions in the nonstochastic model imply that all of the variation in $W_{NN} z_{jN}$ is due to the approximation error, η_{jN}. Therefore, no instrument coming from inside the structural model can be correlated with $W_{NN} z_{jN}$ and uncorrelated with η_{jN}.

To consider new estimation approaches for this model, we introduce empirical counterparts to the structural equation (2) and the reduced-form equation (5) that in addition to the approximation error vector η_{jN} have a random error vector, ε_{jN}, whose elements are independently distributed and vary across all observations ij, so that $E(\varepsilon_{jN}\varepsilon'_{jN}) = diag_{N\times N}(\sigma^2_{\varepsilon,ij})$ in case of heteroskedasticity and $E(\varepsilon_{jN}\varepsilon'_{jN}) = \sigma^2_{\varepsilon} I_{NN}$ in case of homoskedasticity. We assume that the model's approximation error η_{jN} and the random error term ε_{jN} are fully independent of each other.

After adding $l_N + c_N$ on both sides of the equation and introducing the two-component error term $u_{jN} = \eta_{jN} + \varepsilon_{jN}$, we now have for the structural equation

$$\tilde{z}_{jN} \equiv z_{jN} + l_N + c_N \quad = \quad \alpha W_{NN} z_{jN} + \alpha(l_N + c_N) + \alpha(I_{NN} - W_{NN}) t_{jN} + u_{jN}. \tag{7}$$

Similarly, we can write the stochastic counterpart to the reduced form (5) as

$$\tilde{z}_{jN} \quad = \quad \alpha(I_{NN} - W_{NN})(c_N + t_{jN}) + \tilde{u}_{jN}, \tag{8}$$

where $\tilde{u}_{jN} = \frac{1}{1-\alpha}\eta_{jN} + \varepsilon_{jN}$. We use equations (7) and (8) to propose two new simple estimators for the linearized gravity model at hand.

Reduced-form OLS

First, as we showed, (1) can be represented by (7), which relies exclusively on network-weighted exogenous variables but not on network-weighted lags of the dependent variable or the disturbances. While omitting a relevant network-weighted average of the dependent variable from the right-hand side of a model usually leads to an omitted variables bias, this is not the case here. Omitting $\alpha W_{NN} z_{jN}$ from the right-hand side of (2) has only two consequences for equation (7): a rescaling of the constant and of the error term in (7) relative to (2). Therefore, the structural model parameters α and γ can be recovered by estimating the linear regression model

$$\tilde{z}_{jN} \quad = \quad \alpha_w \, \ddot{c}_N + \sum_{h=1}^{H} \beta_h \ddot{d}_{h,jN} + \tilde{u}_{jN} \tag{9}$$

by OLS, where $\ddot{c}_N = (I_{NN} - W_{NN})c_N$ and $\ddot{d}_{h,jN} = (I_{NN} - W_{NN})d_{h,jN}$ are the network-weighted exogenous variables. The OLS coefficient on $\ddot{c}$, α_w, estimates α. The coefficients on the (network-weighted) observable variables that parametrize bilateral trade costs, $\ddot{d}_{jN}$, estimate $\beta_h = \alpha\gamma_h$.[6] These composite parameters might be of independent interest. If not, a consistent estimator of γ_h can be obtained as $\hat{\gamma}_h = \hat{\beta}_h/\hat{\alpha}$.

Since the (scaled) approximation error $\tilde{\eta}_{jN}$ varies only across importers j this leads to the regression error terms $\tilde{u}_{ij}$ being correlated for a given importer j: $Cov(\tilde{u}_{ij}, \tilde{u}_{i'j}) \neq 0$ for two exporters i and i'. Thus, inference after estimation of (9) should rely on cluster-robust standard errors clustered by importers.

This proposed OLS estimation with clustered standard errors is a substantially simpler approach than the econometric models employed in BEK, paving the way for a wider use of the proposed linearization. The model can be easily estimated for all countries jointly for the sake of efficiency gains without any violation of the model assumptions. We denote this approach as RF-OLS, for reduced-form OLS.

Control-function OLS

Our second approach is based directly on the structural equation (2), or, more precisely, its empirical counterpart (7). To the extent that the approximation error η_{jN} is correlated with c_N or $(I_{NN} - W_{NN})t_{jN}$, it may be preferable to estimate (7) instead of using RF-OLS. That is, we can estimate

[6] The model can be estimated with a constant by including a regressor $d_{1,jN} = 1$ in the specification of trade costs with β_1 then being the constant.

the structural model parameters from the linear regression model

$$\tilde{z}_{jN} = \alpha_w \tilde{c}_N + \sum_{h=1}^{H} \beta_h \ddot{d}_{h,jN} + \delta W_{NN} z_{jN} + u_{jN} \tag{10}$$

by OLS. In the latter, α_w on the term $\tilde{c}_N = l_N + c_N$ estimates the key structural parameter α, and the coefficients on the regressors $\ddot{d}_{h,jN}$ estimate $\beta_h = \alpha\gamma_h$. The reason for preferring this approach is that $W_{NN}z_{jN}$ depends linearly on η_{jN}, according to (6). Hence, $\delta W_{NN}z_{jN}$ fully controls for the linearized gravity model's approximation error. This implies that $\delta W_{NN}z_{jN}$ is a control function. Its parameter δ absorbs the potential bias from the correlation of the other regressors with η_{jN}. In other words, while the regression error u_{jN} is endogenous in the sense of violating mean independence, $E(u_{jN}|\tilde{c}_N, \ddot{d}_{jN}) \neq 0$, after conditioning on $W_{NN}z_{jN}$ it is rendered exogenous, $E(u_{jN}|\tilde{c}_N, \ddot{d}_{jN}, W_{NN}z_{jN}) = 0$. Because of the term $W_{NN}z_{jN}$'s role in the control function, the parameter δ should not be interpreted as an estimator for α. Finally, for the same reasons as discussed with the RF-OLS approach, inference should rely on importer-clustered standard errors. We denote this approach as CF-OLS, for control-function OLS.

Both of our suggested approaches can also be implemented as generalized linear model (GLM) estimations. That is, instead of using OLS, equations (9) and (10) can also be estimated via Poisson pseudo-likelihood estimation or other GLM procedures (see Santos Silva and Tenreyro, 2006) with appropriate importer-cluster-robust standard errors. For instance, the control-function approach could be based on the exponential of equation (10),

$$\tilde{Z}_{jN} = \exp(\alpha_w \tilde{c}_N + \sum_{h=1}^{H} \beta_h \ddot{d}_{h,jN} + \delta W_{NN} z_{jN}) \nu_{jN}, \tag{11}$$

where $\tilde{Z}_{jN} = \exp(\tilde{z}_{jN})$ and $\nu_{jN} = \exp(u_{jN}) = \exp(\eta_{jN} + \varepsilon_{jN})$, and the parameters $\{\alpha_w, \beta_1, \ldots, \beta_H, \delta\}$ are estimated by Poisson regression of $\tilde{Z}_{jN}$ on $\{\tilde{c}_N, \ddot{d}_{1,jN}, \ldots, \ddot{d}_{H,jN}, W_{NN}z_{jN}\}$.

Whether OLS, Poisson, or some other GLM estimator is preferred depends on the higher-order properties of the stochastic error ε_{ij} (Santos Silva and Tenreyro, 2006). In this paper, we are more interested in how the approximation error η_{jN} affects estimation, and we further explore this numerically through simulation experiments below.

4 Monte Carlo experiments

4.1 Design of experiments

We construct *worlds* of countries and country pairs according to (1) where everything is known to the simulator, while the researcher does not know the parameters on the regressors. We consider two configurations regarding country numbers, $N \in \{30; 60\}$, leading to numbers of country pairs of $N^2 \in \{900; 3,600\}$. This corresponds to typical data situations found in empirical structural work on gravity models (see Eaton and Kortum, 2002; Anderson and van Wincoop, 2003; Balistreri and Hillberry, 2007; Behrens, Ertur and Koch, 2012). For each of these worlds, we consider three configurations $\alpha \in \{-2; -4; -9\}$, which are supported quantitatively by a sizeable body of work (see Arkolakis, Costinot and Rodríguez-Clare, 2012). Hence, there are six parameter configurations. For each of them, we randomly draw $1,000$ independent vectors of bilateral distances with typical element $DIST_{ij}$ and population sizes with typical element L_i from the empirical distribution of these variables as published by the Centre d'Études Prospectives et d'Informations Internationales for $DIST_{ij}$ and by the World Bank's World Development Indicators for L_i (using the year 2007). We parametrize bilateral trade costs as $t_{ij} = dist_{ij}^{\gamma_{dist}}$, where $dist_{ij}$ is the logarithm of $DIST_{ij}$. In line with the robust result of a coefficient on log distance of about $\beta_{dist} = \alpha\gamma_{dist} = -1$ in empirical gravity models, we assume that log distance is related to log trade costs t_{ij} by a parameter of $\gamma_{dist} = -1/\alpha$. Based on the draws for L_i and t_{ij}, the endogenous variables C_i and X_{ij} are solved by contraction-mapping based on (1).

4.2 Features of model variables and the approximation error

Before turning to estimation, it is useful to study some moments and the correlations of key variables in the model across all experiments. For this purpose, we report the averages of an analysis of variance of some key variables in Table 1 and average partial correlation coefficients in Table 2, each of them computed across all 1,000 draws within one of the six parameter configurations in $\{N; \alpha\}$. We first consider here a data generating process (DGP) without stochastic error component ($\varepsilon_{ij} = 0$ for all i, j), so that the model error $u_{ij} = \eta_{ij} + \varepsilon_{ij}$ consists entirely of the approximation error η_{ij}.

Table 1 reports on sums of squares of key model variables. It reports the total variation in each of the variables (row 'total'), as well as a decomposition of the total into variation across exporters (row 'i'), importers (row 'j') and across 'ij', i.e., the bilateral variation (row 'residual'). The table reveals that the (total) variation in the error, $u_{ij} = \eta_{ij}$, is large relative to normalized bilateral exports in logs, z_{ij}. Its size increases with the absolute level of α; i.e., with the distance to the approximation point used by BEK to linearize the model ($\alpha = 0$).

The approximation error varies to a greater degree than log factor costs, c_N, whose variance is the same as that of $\ddot{c}_N = (I_{NN} - W_{NN})c_N$. The relative magnitude of the sum of squares of u_{ij} relative to that of z_{ij} declines as N, the number of countries, rises. The variance of $(I_{NN} - W_{NN})t_{jN}$, with typical element $\ddot{t}_{ij}$, is important relative to that of c_N. But its importance relative to u_{jN} depends on being closer to the approximation point for α. Clearly, while the exporter- and importer-specific components in t_{ij} are symmetric by design (log-distance is symmetric), those of $\ddot{t}_{ij}$ are not. The pair-specific component of $\ddot{t}_{ij}$ naturally dominates the country-specific ones. Finally, as was clear from the theoretical derivations from the previous section, the variation in u_{ij} is purely importer-specific. This is because BEK's approximation is about an importer-specific term, the log ideal consumer-price index.

Table 2 shows that there is a perfect correlation between the elements of $W_{NN}z_{jN}$ and the ones of the error, u_{jN}, consistent with equation (6) and the fact that in this case $u_{jN} = \eta_{jN}$. There is some small correlation between $\ddot{t}_{jN} = (I_{NN} - W_{NN})t_{jN}$ and u_{jN}. This means that in estimations where $\ddot{t}_{jN}$ is a regressor, the coefficient on $\ddot{t}_{jN}$ may exhibit some bias unless we condition on $W_{NN}z_{jN}$ (which means conditioning on η_{jN}, as mentioned before). This problem becomes more pertinent if the approximation error is larger, which is the case with a bigger absolute value of α.

Figure 1 visualizes the relationships in Table 2 based on one specific random draw for $N = 30$ and $\alpha = -4$. There are four general insights from an inspection of Figure 1 in conjunction with Table 2. First, the upper left panel of the figure documents that $W_{NN}z_{jN}$ is indeed perfectly correlated with $u_{jN} = \eta_{jN}$ as suggested by equation (6). Second, all of the panels in Figure 1 illustrate the block structure of u_{jN} which means it is not independently and identically distributed and which motivates the need to use clustered standard errors for inference. Third, while the correlation between u_{jN} and the other right-hand side model variables is weak on average, it may be stronger depending on the specific configuration of trade costs (t_{jN}) and population size (W_{NN}). From Table 2 we know that the risk of correlation between model variables and u_{jN} is higher for $\ddot{t}_{jN}$ than for c_N. Figure 1, for instance, illustrates a case where $(I_{NN} - W_{NN})t_{jN}$ is negatively and $(\alpha - 1)(l_N + c_N) + \alpha(I_{NN} - W_{NN})t_{jN}$ is positively correlated with u_{jN}. In such a case, we would expect the estimated parameter on $\ddot{t}_{jN}$ to be biased if we do not address the endogeneity with a control function. Altogether we would expect a larger root-mean-squared error for the estimated parameter on this variable than on c_N or $\ddot{c}_N$, unless one controls for u_{jN}.

4.3 Parameter estimation

We compare the estimation of the linearized gravity network model via our two approaches RF-OLS and CF-OLS. To benchmark these estimators, we also compare them to a structural estimation of the original, non-linearized gravity model through an iterative least squares procedure. For this procedure, the dependent variable can also be defined as $\tilde{z}_{jN} = z_{jN} + l_N + c_N$, where z_{jN} are normalized trade flows, which is the same as in the proposed RF-OLS and CF-OLS procedures. Taking the log of (1) and adding the stochastic error vector ε_{jN}, the structural model results in

$$\tilde{z}_{jN} = \alpha_0 + \alpha_c \tilde{c}_N + \alpha_t t_{jN} - m_{jN} + \varepsilon_{jN}, \tag{SILS}$$

where $\tilde{c}_N = l_N + c_N$ and the log-nonlinear term is defined as $m_{jN} \equiv \ln(\sum_{k=1}^{N} L_k C_k^{\alpha} T_{kj}^{\alpha})$. The model can be estimated by structural iterative least squares (cf. Anderson and van Wincoop, 2003), which we denote by SILS. In our implementation of SILS, for an initial guess of $\widehat{m}_{jN}$, the coefficients appearing in equation (SILS) can be estimated by an OLS regression of $\tilde{z}_{jN} + \widehat{m}_{jN}$ on a constant, $\tilde{c}_N$, and t_{jN}. These coefficients can be used to obtain an updated estimate of $\widehat{m}_{jN}$, which in turn can be used to perform an updated OLS regression. These steps are iterated until the values of the estimated coefficients converge.

Note that for (SILS) there is no approximation error term η_{jN}, since no approximation has been applied; this is the true non-linearized structural model. And since in our first DGP there is also no random error (that is, $\varepsilon_{jN} = 0$), SILS in this case is an algorithm to *solve* for the model parameters, rather than to estimate them.

The two estimators of the linearized gravity equation which leads to a network model are based on the following estimating equations

$$\tilde{z}_{jN} = \alpha_0 + \alpha_c \tilde{c}_N + \alpha_t \ddot{t}_{jN} + \delta W_{NN} z_{jN} + u_{jN}, \tag{CF-OLS}$$

$$\tilde{z}_{jN} = \alpha_0 + \alpha_c \ddot{c}_N + \alpha_t \ddot{t}_{jN} + \tilde{u}_{jN}, \tag{RF-OLS}$$

which, as discussed in Section 2, can be estimated by simple OLS. For all three models—(SILS), (CF-OLS) and (RF-OLS)—we only present an unconstrained parameter-estimation version each, which does not enforce that $\alpha_c = \alpha$ and $\alpha_t = \alpha\gamma_{dist}$ are identical due to the chosen parametrization. We do so to mimic the situation of an empirical researcher who does not observe t_{ij} but only $dist_{ij}$. The estimated parameters $\{\hat{\alpha}_c; \hat{\alpha}_t\}$ should be close to the true α, especially, when being based on models (SILS) or (CF-OLS). While in the latter model there is an approximation error, we saw in Section 3 that the control function $W_{NN} z_{jN}$ accounts *fully* for it (that is, not just in expectation, as is typically the case with control function approaches; see, e.g., Wooldridge, 2015).

Apart from a data generating process (DGP) where the structural nonlinear model (SILS) is true, we consider a second DGP with an additional stochastic error term ε_{jN}. Specifically, we specify the random error as $\varepsilon_{ij} \overset{IID}{\sim} N(0, \sigma_\varepsilon^2)$. We calibrate σ_ε^2 so that, in each experiment, the explanatory power as measured by the R^2 is 80% ($= (1 - \sigma_\varepsilon^2/\sigma_{\tilde{z}^*}^2) \times 100\%$), which is representative of a vast amount of empirical work on gravity models. The term ε_{ij} adds stochastics in a narrow sense which provides for a residual with Models (SILS) and (CF-OLS) and one beyond the approximation (or linearization) error in Model (RF-OLS).

We report on the average bias and root-mean-squared error (RMSE) in percent of the true α across all draws per configuration of $\{N; \alpha\}$ in Tables 3 and 4. Both tables are organized in three by two blocks. Each horizontal block contains estimates for the models (SILS), (CF-OLS) and (RF-OLS) for the cases $N = \{30; 60\}$. Vertically, we have three blocks corresponding to $\alpha = \{-2; -4; -9\}$. For each of the six blocks, we report on the estimated structural parameters $\hat{\alpha}_c$ and $\hat{\alpha}_t$; as well as $\hat{\delta}$ for CF-OLS, the coefficient on the control function.

Table 3 reports the results for the DGP without ε_{ij}; that is, only with the approximation error η_{ij} present: $u_{jN} = \eta_{jN}$. In the absence of ε_{ij}, both models (SILS) and (CF-OLS) correspond to the true one so that both the bias and the RMSE for $\{\hat{\alpha}_c; \hat{\alpha}_c\}$ in percent are zero. That is, the CF-OLS estimator of the linearized gravity model is optimal in the sense of suffering zero efficiency loss due to the linearization. Recall that conditioning on $W_{NN} z_{jN}$ means conditioning on η_{jN}, according to (6). Thus, all the linearization bias is picked up by the coefficient $\hat{\delta}$. Only the RF-OLS approach has an estimation residual and therefore a non-zero variance under this DGP. The RF-OLS estimates of α exhibit some bias, but both bias and RMSE are relatively small. For instance, the largest bias in absolute value over all DGP settings and parameters is only -1.21 percent. Further, as expected, both the bias and the RMSE of $\{\hat{\alpha}_c; \hat{\alpha}_t\}$ in percent decline as the number of countries rises. As the number of countries increases, it has been shown that the need for controlling for general equilibrium effects and nonlinear trade-cost effects as captured by m_{jN} in (SILS) also declines (see Egger and Staub, 2016).

Table 4 depicts results for the DGP with both approximation error and stochastic error: $u_{jN} = \eta_{jN} + \varepsilon_{jN}$. The biases for all three estimators are very small, with the maximum bias in absolute value across all parameter configurations and estimators being 1 percent (for RF-OLS in the DGP with $\alpha = -2$ and $N = 30$). The simple CF-OLS estimator achieves RMSEs that are not much larger than those of the more demanding iterative SILS procedure. Even the yet simpler RF-OLS, while exhibiting the highest RMSE of the three estimators, does quite well by this performance measure.

Finally, in Table 5, we present results where all estimators are based on exponentiated equations and a Poisson (pseudo)-likelihood. The estimation quality deteriorates somewhat across all approaches in that case. For SIP and CF-Pois (the Poisson versions of SILS and CF-OLS), the biases continue to be small, with the maximum absolute bias for CF-Pois being about 2 percent. RMSEs are also only moderately higher. With ε_{ij} being homoscedastic normal, OLS estimation in logs is efficient, so these results are expected. The reduced-form approach is not only dependent on ε_{ij} but also on η_{ij}, which is not only non-normal, but also not fully mean-independent of the regressors. The results in the table show that, when exponentiated, the endogeneity is aggravated and RF-Pois suffers from larger absolute biases of up to 9.28 percent. From the perspective of the linearization error η_{ij}, this speaks for using the reduced-form approach in logs (RF-OLS) rather than in exponentiated form (RF-Pois). In contrast, for the control-function approach the properties of the linearization error are of little relevance, and the choice of CF-OLS or CF-Pois can be based entirely on other considerations, such as the properties of the stochastic error.

5 Conclusions

This paper sheds light on the nature of structural linearized gravity models involving an endogenous network-weighted lag – other countries' population-share – of bilateral trade flows as developed in Behrens, Ertur and Koch (2012). We demonstrate that the properties of the network model is such that it can be estimated without any use of network-econometric tools. Exporter-population-share-weighted log bilateral exports on the right-hand side of the models serve as a control function for the approximation error of the linearization, and this variable can be included without specific treatment (i.e., ignoring its endogeneity). These results should please the applied researcher, since estimation of such linearized models only involves OLS (on log-transformed trade flows) with clustered standard errors at the level of importers.

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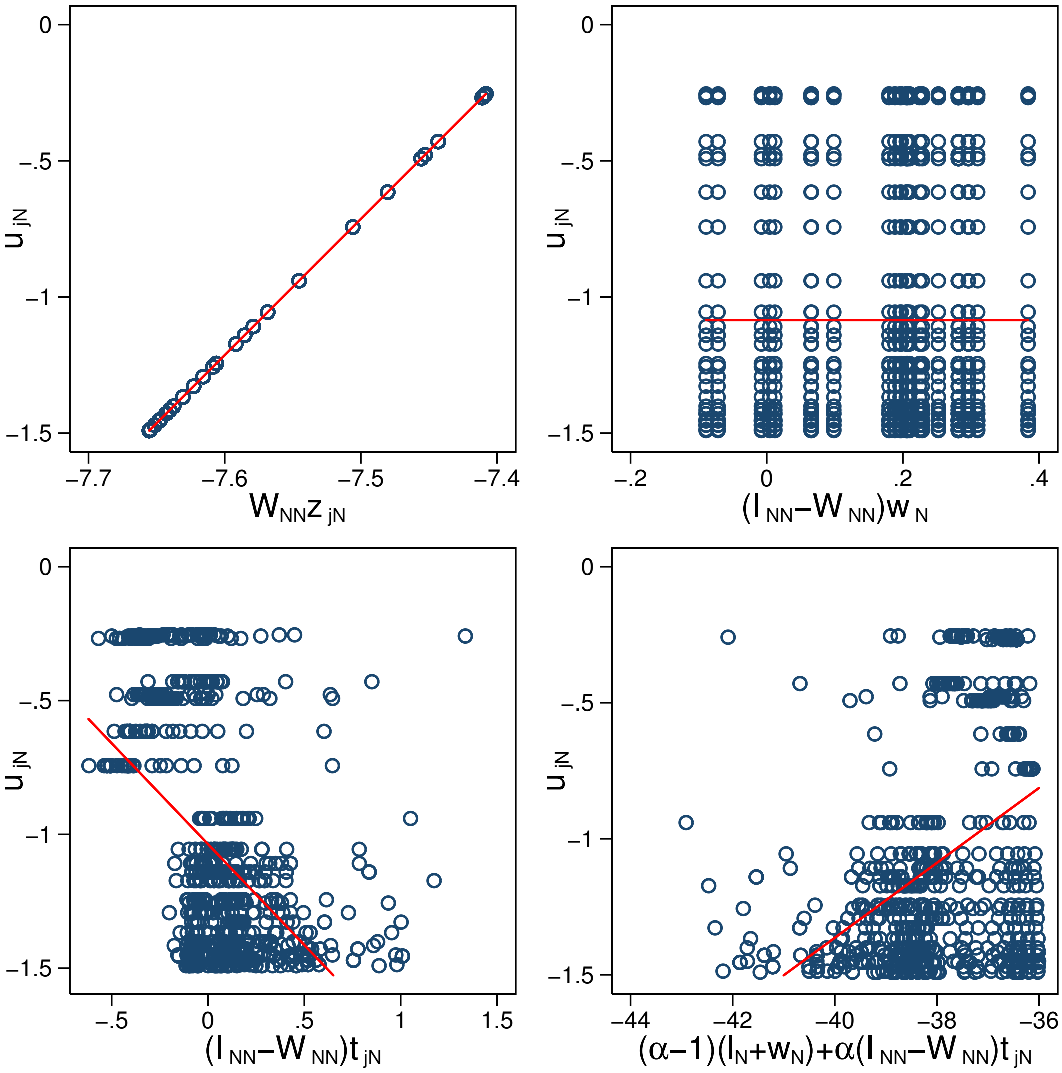
Tables and figures

Figure 1: Scatterplot and linear fit of approximation error ($u_{jN} = \eta_{jN}$) and right-hand side variables from a random draw of the DGP with $\alpha = -4$ for $N = 30$



***Notes*:** The four panels of the figure display scatterplots of data obtained from one random draw of the DGP with $\alpha = -4$ for 30 countries (900 observations). The red line represents the fit from a linear regression.

Table 1: Analysis of variance for key model variables (mean sums of squares over 1,000 replications)

	$N = 30$				$N = 60$			
SS	z_{ij}	u_{ij}	c_i	$\ddot{t}_{ij}$	z_{ij}	u_{ij}	c_i	$\ddot{t}_{ij}$
				$\alpha = -2$				
i (exporter)	112.26	0.00	14.99	20.47	370.46	0.00	52.21	77.74
j (importer)	112.26	68.40	0.00	29.39	370.46	171.00	0.00	95.89
residual	738.85	0.00	0.00	184.71	2398.63	0.00	0.00	599.66
total	963.38	68.40	14.99	234.58	3139.55	171.00	52.21	773.29
				$\alpha = -4$				
i (exporter)	127.93	0.00	6.01	5.11	416.30	0.00	20.81	19.47
j (importer)	127.93	205.19	0.00	7.47	416.30	519.76	0.00	24.03
residual	737.95	0.00	0.00	46.12	2400.35	0.00	0.00	150.02
total	993.82	205.19	6.01	58.70	3232.95	519.76	20.81	193.52
				$\alpha = -9$				
i (exporter)	139.89	0.00	1.63	1.01	458.94	0.00	5.64	3.84
j (importer)	139.89	949.64	0.00	1.48	458.94	2481.73	0.00	4.78
residual	737.95	0.00	0.00	9.11	2398.50	0.00	0.00	29.61
total	1017.72	949.64	1.63	11.60	3316.38	2481.73	5.64	38.23

Notes: SS refers to sum of squares. $\ddot{t}_{ij} \equiv t_{ij} - \sum_i \frac{L_i}{L} t_{ij}$ is a typical element of $(I_{NN} - W_{NN})t_{jN}$.

Table 2: Partial correlation coefficients of model variables with approximation error $u_{jN} = \eta_{jN}$ (mean and standard deviations over 1,000 replications)

	$N = 30$		$N = 60$	
	Mean	SD	Mean	SD
		$\alpha = -2$		
$W_{NN} z_{jN}$	1.00	0.00	1.00	0.00
$\tilde{c}_N$	-0.00	0.00	0.00	0.00
$\ddot{t}_{jN}$	0.06	0.14	0.06	0.10
		$\alpha = -4$		
$W_{NN} z_{jN}$	1.00	0.00	1.00	0.00
$\tilde{c}_N$	-0.00	0.00	-0.00	0.00
$\ddot{t}_{jN}$	-0.01	0.19	-0.02	0.14
		$\alpha = -9$		
$W_{NN} z_{jN}$	1.00	0.00	1.00	0.00
$\tilde{c}_N$	0.00	0.00	-0.00	0.00
$\ddot{t}_{jN}$	-0.06	0.21	-0.08	0.17

Table 3: Average bias and root mean squared error of estimated model parameters (1,000 replications): DGP with approximation error only: $u_{jN} = \eta_{jN}$

		$N = 30$			$N = 60$		
		SILS	CF-OLS	RF-OLS	SILS	CF-OLS	RF-OLS
				$\alpha = -2$			
$\hat{\alpha}_w$	Bias	-0.00	0.00	-1.21	-0.00	-0.00	-0.76
	RMSE	0.00	0.00	2.05	0.00	0.00	1.28
$\hat{\alpha}_t$	Bias	-0.00	-0.00	-0.79	-0.00	-0.00	-0.60
	RMSE	0.00	0.00	1.56	0.00	0.00	1.05
$\hat{\delta}$	Bias	–	-150.00	–	–	-150.00	–
	RMSE	–	150.00	–	–	150.00	–
				$\alpha = -4$			
$\hat{\alpha}_w$	Bias	-0.00	0.00	-0.81	-0.00	-0.00	-0.36
	RMSE	0.00	0.00	1.67	0.00	0.00	0.99
$\hat{\alpha}_t$	Bias	-0.00	-0.00	-0.13	-0.00	-0.00	0.08
	RMSE	0.00	0.00	1.80	0.00	0.00	1.24
$\hat{\delta}$	Bias	–	-125.00	–	–	-125.00	–
	RMSE	–	125.00	–	–	125.00	–
				$\alpha = -9$			
$\hat{\alpha}_w$	Bias	-0.00	-0.00	-0.60	-0.00	0.00	-0.16
	RMSE	0.00	0.00	1.46	0.00	0.00	0.91
$\hat{\alpha}_t$	Bias	-0.00	0.00	0.40	-0.00	0.00	0.64
	RMSE	0.00	0.00	2.25	0.00	0.00	1.74
$\hat{\delta}$	Bias	–	-111.11	–	–	-111.11	–
	RMSE	–	111.11	–	–	111.11	–

Notes: Columns SILS, CF-OLS, and RF-OLS refer to structural iterative least squares, control-function OLS and reduced-form OLS estimates of models (SILS), (CF-OLS) and (RF-OLS) in Section 4.3. Table entries are average biases and root mean squared errors in percent of the true α.

Table 4: Average bias and root mean squared error of estimated model parameters (1,000 replications): DGP with approximation error and stochastic error: $u_{jN} = \eta_{jN} + \varepsilon_{jN}$

		$N = 30$			$N = 60$		
		SILS	CF-OLS	RF-OLS	SILS	CF-OLS	RF-OLS
				$\alpha = -2$			
$\hat{\alpha}_w$	Bias	0.20	-0.00	-1.00	-0.03	-0.18	-0.80
	RMSE	4.13	4.11	4.49	1.70	1.70	2.10
$\hat{\alpha}_t$	Bias	0.01	-0.07	-0.79	0.01	-0.12	-0.59
	RMSE	1.00	1.11	1.76	0.43	0.59	1.13
$\hat{\delta}$	Bias	–	-128.50	–	–	-128.95	–
	RMSE	–	129.33	–	–	129.51	–
				$\alpha = -4$			
$\hat{\alpha}_w$	Bias	0.20	0.12	-0.61	-0.05	-0.04	-0.41
	RMSE	3.32	3.30	3.62	1.38	1.38	1.68
$\hat{\alpha}_t$	Bias	0.00	0.33	-0.14	0.00	0.24	0.09
	RMSE	1.04	1.37	1.99	0.45	0.77	1.31
$\hat{\delta}$	Bias	–	-114.20	–	–	-114.57	–
	RMSE	–	114.42	–	–	114.72	–
				$\alpha = -9$			
$\hat{\alpha}_w$	Bias	0.15	0.12	-0.45	-0.05	0.03	-0.20
	RMSE	2.94	2.94	3.22	1.22	1.22	1.51
$\hat{\alpha}_t$	Bias	0.00	0.63	0.39	0.00	0.50	0.65
	RMSE	1.06	1.67	2.41	0.46	1.01	1.80
$\hat{\delta}$	Bias	–	-106.46	–	–	-106.70	–
	RMSE	–	106.51	–	–	106.73	–

Notes: Columns SILS, CF-OLS, and RF-OLS refer to structural iterative least squares, control-function OLS and reduced-form OLS estimates of models (SILS), (CF-OLS) and (RF-OLS) in Section 4.3. Table entries are average biases and root mean squared errors in percent of the true α.

Table 5: Poisson PML estimators — Average bias and root mean squared error of estimated model parameters (1,000 replications): DGP with approximation error and stochastic error ($u_{jN} = \eta_{jN} + \varepsilon_{jN}$)

		$N = 30$			$N = 60$		
		SIP	CF-Pois	RF-Pois	SIP	CF-Pois	RF-Pois
			$\alpha = -2$				
$\hat{\alpha}_w$	Bias	0.31	-2.15	-9.28	0.02	-1.94	-7.08
	RMSE	6.69	5.70	13.62	2.65	3.00	9.18
$\hat{\alpha}_t$	Bias	0.07	-1.46	-6.00	0.03	-1.32	-4.49
	RMSE	3.36	2.78	7.90	1.13	1.82	5.27
$\hat{\delta}$	Bias	–	-129.90	–	–	-130.49	–
	RMSE	–	130.60	–	–	130.95	–
			$\alpha = -4$				
$\hat{\alpha}_w$	Bias	0.28	-1.78	-7.92	-0.01	-1.60	-5.99
	RMSE	5.65	4.61	11.61	2.21	2.46	7.91
$\hat{\alpha}_t$	Bias	0.09	-0.37	-4.05	0.02	-0.46	-2.75
	RMSE	3.83	2.86	7.59	1.24	1.49	4.55
$\hat{\delta}$	Bias	–	-115.05	–	–	-115.28	–
	RMSE	–	115.23	–	–	115.41	–
			$\alpha = -9$				
$\hat{\alpha}_w$	Bias	0.24	-1.64	-7.41	-0.02	-1.40	-5.56
	RMSE	5.28	4.12	10.66	1.97	2.17	7.32
$\hat{\alpha}_t$	Bias	0.11	0.39	-2.58	0.02	0.19	-1.36
	RMSE	4.44	3.48	8.08	1.34	1.73	4.80
$\hat{\delta}$	Bias	–	-106.92	–	–	-107.02	–
	RMSE	–	106.95	–	–	107.05	–

Notes: Columns SIP, CF-Pois, and RF-Pois refer to structural iterative Poisson PML, control-function Poisson PML and reduced-form Poisson PML estimates of (exponentiated versions of) models (SILS), (CF-OLS) and (RF-OLS) in Section 4.3. Table entries are average biases and root mean squared errors in percent of the true α.

Appendix. Generic gravity models with constant markups

Arkolakis et al. (2012) demonstrate the generic structure of a host of gravity-model types featuring constant markups, where output prices change in response to trade costs exclusively due to endogenous adjustments in costs but not markups. Models which fall in this class are Dixit-Stiglitz-Krugman type models of monopolistically competitive firms (see, e.g., Bergstrand et al., 2013 for a multi-country gravity version of this type), Ricardian models with perfectly competitive firms (see Eaton and Kortum, 2002), and Armington endowment-economy models (see Anderson and van Wincoop, 2003). All of these models fundamentally adhere to the structure in (1), and what they differ by are only the interpretations of L_i, C_i, and α (the so-called trade elasticity).

While α is directly related to the elasticity of produced varieties—of firms in Dixit-Stiglitz-Krugman models and of countries in Anderson-van-Wincoop models—it measures the production-cost (or productivity) dispersion among the potential producers any country.

L_i is a measure of factor endowments (and firm numbers) in Dixit-Stiglitz-Krugman models, a measure of preference-scaled factor or goods endowments in Anderson-van-Wincoop models, and a measure of average country-level productivity in Eaton-Kortum models. It can generally be obtained when normalizing (dividing) country i's aggregate sales value (in most models GDP) Y_i by C_i.

C_i are the costs per unit of L_i. It can be wages in a Dixit-Stiglitz-Krugman model, where L_i is a country's labor endowment. If L_i is a factor bundle, C_i measures the unit costs of the bundle (e.g., a Cobb-Douglas aggregate of observed factor costs). This is the same in an endowment-economy model, if L_i measures the labor endowment. If L_i is a preference-scaled endowment with goods or a scaled Armington parameter as in Anderson and van Wincoop (2003), C_i measures the unit value or price (per exported unit of good). in Eaton-Kortum models, C_i also measures the variable factor costs per unit of output, as in a Dixit-Stiglitz-Krugman framework.

For all these models, the numerator in the expression of Z_{ij} in equation (1) is log-additive, while the denominator, to be interpreted as the $-\alpha$-scaled log of the ideal consumer price in j, is not log-linear. The latter is in the focus of BEK's linearization. Because all of the mentioned models feature a price index of this form, BEK's linearization is a powerful tool for everyone of them.

All of the mentioned gravity models—with different interpretations of $\{L_i, C_i, \alpha\}$—have the original form for nominal bilateral exports of

$$X_{ij} \quad = \quad \frac{L_i C_i^{\alpha} T_{ij}^{\alpha}}{\sum_{k=1} L_k C_k^{\alpha} T_{kj}^{\alpha}} Y_j. \tag{1}$$

What this equation says is that bilateral purchases or expenditures are proportional to the aggregate level of expenditures in a country. Note that the ratio term appearing in (1) allocates these expenditures across all sources, including the domestic market. After dividing both sides of (1) by Y_j, we arrive at equation (10) in Eaton and Kortum (2002). Moreover, we see that (1) has exactly the structural form given in equations (6) and (7) in Anderson and van Wincoop (2003). And the form is the same as in equation (9) in Bergstrand et al. (2013).[7]

Using $Z_{ij} = X_{ij}/(L_i C_i Y_j)$, we arrive at equation (1) and the corresponding right-hand side of the model given there (we have used the short hand of $L_i C_i = Y_i$, there). This expression is used by Behrens et al. (2012). Log-transforming (1) and linearizing it in the point where $\alpha = 0$, Behrens et al. (2012) arrive at equation (2) in the main text above.

[7] As said, what is called $\{L_i, C_i\}$ here would be $\{T_i c_i\}$ in Eaton and Kortum (2002), it would be $\{\beta_i^{\alpha}, p_i\}$ in Anderson and van Wincoop (2003), and it would be $\{L_i, w_i\}$ with $w_i = Y_i/L_i$ in Bergstrand et al. (2013).