Estimating trade network models without network econometrics

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Abstract

This paper studies structural network gravity models à la Behrens, Ertur and Koch (2012). Such models provide an elegant linearization of nonlinear structural gravity models of international trade, and have a wide range of other applications to bilateral flow data such as investments and migration. In the context of trade, these models have the desirable feature that they account for so-called multilateral resistance (or aggregate price index) terms in a theory-consistent way. Earlier research had proposed applying network-econometric techniques for estimating such models. We exploit the structure of the model to propose simple alternative estimators that do not require any specific network methods, making the structural network model amenable to broader use by practitioners. We show that all structural model parameters can be recovered from a linear OLS regression that uses a properly-weighted average of the dependent variable as a control function. Our control-function approach can also be implemented with simple nonlinear estimators instead of OLS, such as Poisson pseudo maximum likelihood or other generalized linear model estimators.

Keywords: Gravity models; Multilateral resistance terms; Network models.

JEL-codes: F14; C23.

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1 Introduction

The gravity equation—describing aggregate demand for goods or services between any pair of countries—is among the most successful concepts in all of economics (see Leamer and Levinsohn, 1995). Its popularity derives from the fact that it nests a wide variety of isomorphic structural models of aggregate bilateral demand such as endowment-economy, Ricardian, and monopolistic-competition-increasing-returns-to-scale (see Eaton and Kortum, 2002; Anderson and van Wincoop, 2003; Arkolakis, Costinot and Rodríguez-Clare, 2012; Bergstrand, Egger and Larch, 2013; Baltagi, Egger and Pfaffermayr, 2015).

The main reason why gravity equations are rarely estimated in their structural form is that they are highly nonlinear even after taking logarithms. Rather, practitioners use either country fixed effects, which may be inefficient, or theory-based linear approximations in estimation. Behrens, Ertur and Koch (2012, henceforth BEK) introduced such a linearization in a quantity-based version of a constant elasticity of substitution model, where price indices are themselves implicit functions of trade flows. The linearized equilibrium system leads to an econometric specification in which trade flows between two countries depend on trade flows between all other trading partners, thus exhibiting the characteristics of a network-weighted model. To deal with this interdependence among observations, BEK adapt methods from the literature on network econometrics that account for simultaneous, cross-sectionally autoregressive structures. While the model has since been widely adopted, an existing barrier to its further dissemination lies in the network-based estimation, which requires either the availability of instrumental variables or the implementation of computationally intensive maximum likelihood procedures.

In this paper, we propose estimators that exploit the structural form of the same network model to recover the model's structural parameters. In contrast to the previous network-based approach to estimating the model, the proposed estimators are simple OLS (or Poisson PML—pseudo maximum likelihood) estimators. In our preferred proposed approach, the network structure is fully captured through the inclusion of an appropriate regressor that serves as a control function for the endogeneity resulting from the model's network structure. As a result, the proposed estimator is fully efficient in the sense of there not being any efficiency loss due to estimating the linear approximation rather than the true non-linearized model. Constructing this control function is straightforward as it is just a weighted average of the data and does neither require instruments nor additional estimation steps. Therefore, the proposed alternative estimators are easily implementable in practice, providing a low-cost way for practitioners to estimate this popular structural network gravity model, either applied to international trade flows as in the original BEK paper, or applied to other bilateral flow data such as immigration or financial investment.

We examine the properties of the structural gravity model and this linearization as well as the finite sample performance of the estimators in a series of numerical Monte Carlo experiments. The simulation data are obtained by randomly drawing exogenous variables from real-world data and then solving for endogenous variables according to the theory-based structural general equilibrium model. As we show theoretically and confirm in the simulations, the only source of bias in a proposed reduced-form OLS estimation stems from the original linearization of the model. Overall, the simulation results demonstrate that these biases are moderate and shrink towards zero with increasing sample size. Moreover, the preferred new control-function OLS approach does not suffer from this linearization bias and is unbiased in all sample sizes.

We continue with presenting BEK's trade network model in Section 2, and show how this model can be estimated by linear (or generalized linear) methods in Section 3. A numerical assessment of the estimators is provided in Section 4, followed by our conclusions in Section 5.

2 A linearized structural network gravity model of trade

We consider a standard gravity equation derived from utility maximization subject to income constraints, which can be represented for exporter i and importer j as

$$Z_{ij} \equiv \frac{X_{ij}}{Y_i Y_j} = \frac{W_i^{\alpha-1} T_{ij}^{\alpha}}{\sum_{k=1}^{N} L_k W_k^{\alpha} T_{kj}^{\alpha}}, \qquad W_i = L_i^{-1} \sum_{j=1}^{N} X_{ij} = \sum_{j=1}^{N} \frac{W_i^{\alpha} T_{ij}^{\alpha} W_j L_j}{\sum_{k=1}^{N} L_k W_k^{\alpha} T_{kj}^{\alpha}}. \tag{1}$$

Here, Z_{ij} are aggregate bilateral exports, X_{ij}, normalized by exporter and importer GDP, Y_i and Y_j, respectively. The variable W_i represents wages or producer prices, T_{ij} are bilateral trade costs, and L_i is population or size of the labor force (see Arkolakis, Costinot and Rodríguez-Clare, 2012). As mentioned, (1) is compatible with a variety of aggregate bilateral demand models such as endowment-economy, Ricardian, and monopolistic-competition-increasing-returns-to-scale.

Equation (1) shows that the log of Z_{ij}, z_{ij}, is a log-nonlinear function of $\{W_i; T_{ij}; L_i\}$. The key structural parameter $\alpha = (-\infty, 0)$ reflects the partial response of trade with respect to changes in trade costs (see Dixit and Stiglitz, 1977, or Eaton and Kortum, 2002). Through (1), upon choice of a numéraire wage ($W_1 = 1$) and for a given α, the $N-1$ endogenous values of W_i are determined by $N-1$ equations for given (exogenous) values of L_i and T_{ij}^{α}.[1]

BEK take the logarithm of equation (1) and derive a first-order approximation around the point $\alpha = 0$. To provide a compact notation, let us generally use the convention that lower-case letters refer to variables in logarithms and refer to N-size vectors and square matrices by subscripts N and NN, respectively. Letting the world population be denoted by $L \equiv \sum_{k=1}^{N} L_k$, define the following matrices:

(i) W_{NN} which in column j contains the same elements L_j/L in all the rows,

[1] In BEK's parametrization the size of countries is parameterized by the endowment L_i. In an Eaton-Kortum Ricardian economy this would be productivity, and in an Armington economy it would be a preference shifter. For the purpose of the arguments in this paper, these differences are only semantic.

(ii) $D_{NN} = diag_N(L_j/L)$ which contains the diagonal elements of W_{NN},

(iii) $\tilde{W}_{NN} \equiv W_{NN} - D_{NN}$, and

(iv) the identity matrix I_{NN}.

Then, stacking all observations across exporters i for a given importer j in $z_{jN} = (z_{ij})$, $l_N = (l)$, $0w_N = (w_i)$ and $t_{jN} = (t_{ij})$, BEK arrive at the log-transformed and linearized counterpart to (1):

$$z_{jN} \quad = \quad \alpha W_{NN} z_{jN} + (\alpha - 1)(l_N + w_N) + \alpha(I_{NN} - W_{NN}) t_{jN} + u_{jN}, \tag{2}$$

where u_{jN} is the approximation error due to linearization, which only varies across j but not i.[2,3] In its role as coefficient on $W_{NN} z_{jN}$, BEK refer to α in equation (2) as the *autoregressive interaction coefficient* which captures the interdependence across trade flows and provides an intuitive network measure of 'spatial competition.' It is customary in empirical work to further specify $t_{ij} = \sum_{h=1}^{H} \gamma_h d_{h,ij}$, where $d_{h,ij}$ are observable variables in logs such as bilateral log-distance. What is then estimated on $d_{h,ij}$ are the compound parameters $\alpha\gamma_h$.

The reduced form that directly corresponds to (2) is

$$z_{jN} \quad = \quad (I_{NN} - \alpha W_{NN})^{-1} [(\alpha - 1)(l_N + w_N) + \alpha(I_{NN} - W_{NN}) t_{jN} + u_{jN}]. \tag{3}$$

Existence of the latter requires $(I_{NN} - \alpha W_{NN})$ to have finite elements and to be invertible independent of the number of countries N, a property which we will discuss below. Continuing from (2), BEK reformulate the model to obtain alternate structural and reduced forms

$$\begin{aligned} z_{jN} \; = \; & (I_{NN} - \alpha D_{NN})^{-1} \\ & \times [\alpha \tilde{W}_{NN} z_{jN} + (\alpha - 1)(l_N + w_N) + \alpha(I_{NN} - W_{NN}) t_{jN} + u_{jN}], \end{aligned} \tag{4}$$

$$\begin{aligned} z_{jN} \; = \; & [I_{NN} - \alpha(I_{NN} - \alpha D_{NN})^{-1} \tilde{W}_{NN}]^{-1} (I_{NN} - \alpha D_{NN})^{-1} \\ & \times [(\alpha - 1)(l_N + w_N) + \alpha(I_{NN} - W_{NN}) t_{jN} + u_{jN}], \end{aligned} \tag{5}$$

respectively. The presence of $(I_{NN} - \alpha D_{NN})^{-1}$ in (4) makes the model nonlinear in α. BEK propose estimating a model with right-hand side variables $\{\tilde{W}_{NN} z_{jN}; (l_N + w_N); (I_{NN} - W_{NN}) t_{jN}\}$,

[2]This is equation (11) in BEK. The notation in BEK uses the parametrization $\alpha \equiv 1 - \sigma$ and stacks further across all j.

[3]After defining $Y \equiv \sum_{i=1}^{N} Y_i$, at the approximation point $\alpha = 0$ of the model, we obtain $X_{ij} = \frac{L_i Y_j}{L}$ and $Y_i = \sum_{j=1}^{N} X_{ij} = \frac{L_i Y}{L}$, which implies wage equalization, $W_i = W$. Choosing the wage as the numéraire, we obtain $X_{ij} = \frac{L_i L_j}{L}$. Then, trade costs are irrelevant, and the variance of log bilateral exports, $x_{ij} = \ln X_{ij}$, is fully determined by the variation in exporter- and importer-specific log labor endowments (or population) across countries or regions i and j.

which they denote a heterogeneous coefficients model since the coefficients on these variables are proportional to $(1 - \alpha L_i/L)^{-1}$. Such a model is "econometrically complex to handle" (BEK). Therefore, BEK proposed estimating a model that ignores the heterogeneity and constrains the coefficients to be constant. This approach is inconsistent, but might provide a useful first benchmark. A second approach proposed by BEK is to estimate the model for each i separately. This approach is consistent, but it is inefficient.

3 Properties of the linearized network model and novel insights

The alternative estimators of the linearized network gravity model that we propose are based on the identity of the reduced forms (5) and (3), which suggests that approaches other than the reformulation in (4) can be used to estimate the network-weighted model. To see this, notice that

$$[I_{NN} - \alpha(I_{NN} - \alpha D_{NN})^{-1}\tilde{W}_{NN}]^{-1}(I_{NN} - \alpha D_{NN})^{-1} = (I_{NN} - \alpha W_{NN})^{-1}.$$

The matrix $(I_{NN} - \alpha W_{NN})$ is invertible for any finite $\alpha \neq 1$ as then it has full rank. As $\alpha = 1$ is outside the theoretically admissible parameter space, invertibility is not a concern. Since W_{NN} is idempotent so that $W_{NN}^2 = W_{NN}$, the inverse $(I_{NN} - \alpha W_{NN})^{-1} = I_{NN} + \frac{\alpha}{1-\alpha}W_{NN}$, and $W_{NN}(I_{NN} - \alpha W_{NN})^{-1} = \frac{1}{1-\alpha}W_{NN}$.

Moreover, since neither u_{jN} nor l_N vary across exporters i, $W_{NN}u_{jN} = u_{jN}$ and $W_{NN}l_N = l_N$. Therefore,

$$W_{NN}z_{jN} \quad = \quad -W_{NN}(l_N + w_N) + \frac{1}{1-\alpha}u_{jN}, \tag{6}$$

where $-W_{NN}(l_N + w_N)$ is a constant. This result has important consequences for the econometric approaches to estimate this network gravity model proposed in the literature which are based on two-stage least-squares estimators in the spirit of Kelejian and Prucha (1998), Lee (2003), or Kelejian, Prucha and Yuzefovich (2004). Two-stage least-squares approaches require instruments for $W_{NN}z_{jN}$ which satisfy the exogeneity assumption. Equation (6) shows that there are no suitable instruments for $W_{NN}z_{jN}$ in this model. The nature of $\{W_{NN}, l_N, w_N, u_{jN}\}$ and the parameter restrictions in the model imply that all of the variation in $W_{NN}z_{jN}$ is due to the approximation error, u_{jN}. Therefore, no instrument can be correlated with $W_{NN}z_{jN}$ and uncorrelated with u_{jN}.

Finally, replacing $W_{NN}z_{jN}$ in (2) by the right-hand side in (6) and adding $l_N + w_N$ on both sides of the equation results in

$$\tilde{z}_{jN} \equiv z_{jN} + l_N + w_N \quad = \quad \alpha(I_{NN} - W_{NN})(w_N + t_{jN}) + \frac{1}{1-\alpha}u_{jN}, \tag{7}$$

an alternative form for the linearized model. We use equations (6) and (7) to propose two new simple estimation approaches for the linearized network model.

Reduced-form OLS

First, as we showed, (1) can be represented by (7), which relies exclusively on network-weighted exogenous variables, but not on network-weighted lags of the dependent variable or the disturbances. While omitting a relevant network-weighted average of the dependent variable from the right-hand side of a model usually leads to an omitted variables bias, this is not the case here. Omitting $\alpha W_{NN} z_{jN}$ from the right-hand side of (2) has only two consequences for equation (7): a rescaling of the constant and of the error term in (7) relative to (2). Therefore, the structural model parameters α and γ can be recovered by estimating the linear regression model

$$\tilde{z}_{jN} \quad = \quad \alpha_w \, \ddot{w}_N + \sum_{h=1}^{H} \beta_h \ddot{d}_{h,jN} + \tilde{u}_{jN} \tag{8}$$

by OLS, where $\ddot{w}_N = (I_{NN} - W_{NN}) w_N$, $\ddot{d}_{h,jN} = (I_{NN} - W_{NN}) d_{h,jN}$, and $\tilde{u}_{jN} = u_{jN}/(1-\alpha)$. The OLS coefficient on $\ddot{w}$, α_w, estimates α. The coefficients on the (network-weighted) observable variables that parametrise bilateral trade costs, $\ddot{d}_{jN}$, estimate $\beta_h = \alpha\gamma_h$.[4] These composite parameters might be of independent interest. If not, a consistent estimator of γ_h can be obtained as $\hat{\gamma}_h = \hat{\beta}_h/\hat{\alpha}$.

Since the approximation error u_{jN}—and therefore also the regression error $\tilde{u}_j N$—varies only across importers j and exclusively depends on exogenous model variables and parameters, it appears natural to specify it as heteroskedastic and clustered by exporting country i. Hence, inference after estimation of (8) should rely on cluster-robust standard errors. This is a substantially simpler approach than the econometric models employed in BEK, paving the way for a wider use of the proposed linearization. The model can be estimated for all countries jointly for the sake of efficiency gains without any violation of the model assumptions. We denote this approach as RF-OLS, for reduced-form OLS.

Control-function OLS

Our second approach is based directly on equation (2). To the extent that the approximation error u_{jN} is correlated with w_N or $(I_{NN} - W_{NN}) t_{jN}$, it may be preferable to estimate (2) instead of using RF-OLS. That is to estimate the structural model parameters from the linear regression model

$$\tilde{z}_{jN} \quad = \quad \alpha_w \tilde{w}_N + \sum_{h=1}^{H} \beta_h \ddot{d}_{h,jN} + \delta W_{NN} z_{jN} + u_{jN}, \tag{9}$$

by OLS, where $\tilde{w}_N = l_N + w_N$ and, similar to the case of RF-OLS, the coefficient α_w on the regressor $\tilde{w}_N$ estimates the key structural parameter α, and the coefficients on the regressors $\ddot{d}_{h,jN}$ estimate $\beta_h = \alpha\gamma_h$. The reason for preferring this approach is that $W_{NN} z_{jN}$ depends linearly on

[4] The model can be estimated with a constant by including a regressor $d_{1,jN} = 1$ in the specification of trade costs with β_1 then being the constant.

u_{jN} according to (6) and, hence, fully controls for the linearized gravity model's approximation error. This implies that $W_{NN}z_{jN}$ is a control function. Its parameter absorbs the potential bias from the correlation of the other regressors with u_{jN}. Because of this, the parameter δ on $W_{NN}z_{jN}$ should not be interpreted as an estimator for α. As in the previous case, inference should rely on exporter-clustered standard errors. We denote this approach as CF-OLS, for control-function OLS.

Both of our suggested approaches can also be implemented as generalized linear model (GLM) estimations. That is, instead of using OLS, equations (8) and (9) can also be estimated via Poisson pseudo-likelihood estimation or other GLM procedures (see Santos Silva and Tenreyro, 2006) with appropriate exporter-cluster-robust standard errors. For instance, the control-function approach could be based on the exponential of equation (9),

$$\tilde{Z}_{jN} = \exp(\alpha_w \tilde{w}_N + \sum_{h=1}^{H} \beta_h \ddot{d}_{h,jN} + \delta W_{NN} z_{jN}) \eta_{jN}, \tag{10}$$

where $\tilde{Z}_{jN} = \exp(\tilde{z}_{jN})$ and $\eta_{jN} = \exp(u_{jN})$, and the parameters $\{\alpha_w, \beta_1, \ldots, \beta_H, \delta\}$ are estimated by Poisson regression of $\tilde{Z}_{jN}$ on $\{\tilde{w}_N, \ddot{d}_{1,jN}, \ldots, \ddot{d}_{H,jN}, W_{NN}z_{jN}\}$.

4 Monte Carlo experiments

4.1 Design of experiments

We construct *worlds* of countries and country pairs according to (1) where everything is known to the simulator, while the researcher does not know the parameters on the regressors. We consider two configurations regarding country numbers, $N \in \{30; 60\}$, leading to numbers of country pairs of $N^2 \in \{900; 1,600\}$. This corresponds to typical data situations found in empirical structural work on gravity models (see Eaton and Kortum, 2002; Anderson and van Wincoop, 2003; Balistreri and Hillberry, 2007; Behrens, Ertur and Koch, 2012). For each of these worlds, we consider three configurations $\alpha \in \{-2; -4; -9\}$, which are supported quantitatively by a sizeable body of work (see Arkolakis, Costinot and Rodríguez-Clare, 2012). Hence, there are six parameter configurations. For each of them, we randomly draw $1,000$ independent vectors of bilateral distances with typical element $DIST_{ij}$ and population sizes with typical element L_i from the empirical distribution of these variables as published by the Centre d'Études Prospectives et d'Informations Internationales for $DIST_{ij}$ and by the World Bank's World Development Indicators for L_i (using the year 2007). We parametrise bilateral trade costs as $t_{ij} = dist_{ij}^{\gamma_{dist}}$, where $dist_{ij}$ is the logarithm of $DIST_{ij}$. In line with the robust result of a coefficient on log distance of about $\beta_{dist} = \alpha\gamma_{dist} = -1$ in empirical gravity models, we assume that log distance is related to log trade costs t_{ij} by a parameter of $\gamma_{dist} = -1/\alpha$. Based on the draws for L_i and t_{ij}, the endogenous variables W_i and X_{ij} are solved by contraction-mapping based on (1).

4.2 Features of model variables and the approximation error

Before turning to estimation, it is useful to study some moments and the correlations of key variables in the model across all experiments. For this purpose, we report on the averages of an analysis of variance of some key variables in Table 1 and on average partial correlation coefficients in Table 2, each of them computed across all 1,000 draws within one of the six parameter configurations in $\{N;\alpha\}$.

Table 1 reports on sums of squares of key model variables. It reports the total variation in each of the variables (row 'total'), as well as a decomposition of the total into variation across exporters (row 'i'), importers (row 'j') and across 'ij', i.e., the bilateral variation (row 'residual'). The table reveals that the (total) variation in the approximation error, u_{ij}, is large relative to normalized bilateral exports in logs, z_{ij}. Its size rises with the absolute level of α; i.e., with the distance to the approximation point used by BEK to linearize the model ($\alpha = 0$).

The approximation error varies to a greater degree than log wages, w_N, whose variance is the same as that of $\ddot{w}_N = (I_{NN} - W_{NN})w_N$. The relative magnitude of the sum of squares of u_{ij} relative to that of z_{ij} declines as N, the number of countries, rises. The variance of $(I_{NN} - W_{NN})t_{jN}$, with typical element $\ddot{t}_{ij}$, is important relative to that of w_N. But its importance relative to u_{jN} depends on being closer to the approximation point for α. Clearly, while the exporter- and importer-specific components in t_{ij} are symmetric by design (log-distance is symmetric), those of $\ddot{t}_{ij}$ are not. The pair-specific component of $\ddot{t}_{ij}$ naturally dominates the country-specific ones. Finally, as was clear from the theoretical derivations from the previous section, the variation in u_{ij} is purely importer-specific. This is because BEK's approximation is about an importer-specific term, the log ideal consumer-price index.

Table 2 shows that there is a perfect correlation between the elements of $W_{NN}z_{jN}$ and the ones of the approximation error, u_{jN}, consistent with equation (6). There is some small correlation between $\ddot{t}_{jN} = (I_{NN} - W_{NN})t_{jN}$ and u_{jN}. This means that in estimations where $\ddot{t}_{jN}$ is a regressor, the coefficient on $\ddot{t}_{jN}$ may exhibit some bias unless we condition on $W_{NN}z_{jN}$ (which means conditioning on u_{jN}, as mentioned before). This problem becomes more pertinent if the approximation error is larger, which is the case with a bigger absolute value of α.

Figure 1 visualizes the relationships in Table 2 based on one specific random draw for $N = 30$ and $\alpha = -4$. There are four general insights from an inspection of Figure 1 in conjunction with Table 2. First, the upper left panel of the figure documents that $W_{NN}z_{jN}$ is indeed perfectly correlated with u_{jN} as suggested by equation (6). Second, all of the panels in Figure 1 illustrate the block structure of u_{jN} which means it is not independently and identically distributed. Third, while the correlation between u_{jN} and the other right-hand side model variables is weak on average, it may be stronger depending on the specific configuration of trade costs (t_{jN}) and population size (W_{NN}). From Table 2 we know that the risk of correlation between model variables and u_{jN} is higher for $\ddot{t}_{jN}$ than for w_N. Figure 1, for instance, illustrates a case where $(I_{NN} - W_{NN})t_{jN}$ is negatively and

$(\alpha-1)(l_N+w_N)+\alpha(I_{NN}-W_{NN})t_{jN}$ is positively correlated with u_{jN}. In such a case, we would expect the estimated parameter on $\ddot{t}_{jN}$ to be biased if we do not address the endogeneity with a control function. Altogether we would expect a larger root-mean-squared error for the estimated parameter on this variable than on w_N or $\ddot{w}_N$, unless one controls for u_{jN}.

4.3 Parameter estimation

We compare the estimation of the linearized network gravity model via our two approaches RF-OLS and CF-OLS. To benchmark these estimators, we also compare them to a structural estimation of the original, non-linearized gravity model through an iterative least squares procedure. For this procedure, the dependent variable can also be defined as $\tilde{z}_{jN} = z_{jN} + l_N + w_N$, where z_{jN} are normalized trade flows, which is the same as in the proposed RF-OLS and CF-OLS procedures. Taking the log of (1), the structural model results in

$$\tilde{z}_{jN} = \alpha_0 + \alpha_w \tilde{w}_N + \alpha_t t_{jN} - M_{jN}, \tag{SILS}$$

where $\tilde{w}_N = l_N + w_N$ and the log-nonlinear term is defined as $M_{jN} \equiv \left(\ln(\sum_{k=1}^{N} L_k W_k^{\alpha} T_{kj}^{\alpha})\right)$. The model can be estimated by structural iterative least squares (cf. Anderson and van Wincoop, 2003), which we denote by SILS. In our implementation of SILS, for an initial guess of $\widehat{M}_{jN}$, the coefficients appearing in equation (SILS) can be estimated by an OLS regression of $\tilde{z}_{jN} + \widehat{M}_{jN}$ on a constant, $\tilde{w}_N$, and t_{jN}. These coefficients can be used to obtain an updated estimate of $\widehat{M}_{jN}$, which in turn can be used to perform an updated OLS regression. These steps are iterated until the values of the estimated coefficients converge (do not change anymore beyond some small threshold). Note that there is no error term in this model, since no approximation has been applied to it (and we did not add a stochastic term beyond the structural model). In that sense, SILS here is an algorithm to solve for the model parameters, rather than to estimate them.

The two estimators of the linearized gravity equation which leads to a network model are based on the following estimating equations

$$\tilde{z}_{jN} = \alpha_0 + \alpha_w \tilde{w}_N + \alpha_t \ddot{t}_{jN} + \delta W_{NN} z_{jN} + u_{jN}, \tag{CF-OLS}$$

$$\tilde{z}_{jN} = \alpha_0 + \alpha_w \ddot{w}_N + \alpha_t \ddot{t}_{jN} + \tilde{u}_{jN}, \tag{RF-OLS}$$

which, as discussed in Section 2, can be estimated by simple OLS. For all three models—(SILS), (CF-OLS) and (RF-OLS)—we only present an unconstrained parameter-estimation version each, which does not enforce that $\alpha_w = \alpha$ and $\alpha_t = \alpha\gamma_{dist}$ are identical due to the chosen parametrization. We do so to mimic the situation of an empirical researcher who does not observe t_{ij} but only $dist_{ij}$. The estimated parameters $\{\hat{\alpha}_w; \hat{\alpha}_t\}$ should be close to the true α, especially, when being based on models (SILS) or (CF-OLS). While in the latter model there is an approximation error, we saw in Section 2 that the control function $W_{NN}z_{jN}$ accounts *fully* for it (that is, not just in expectation, as is typically the case with control function approaches; see, e.g., Wooldridge, 2015).

Apart from a data generating process (DGP) where the structural nonlinear model (SILS) is true, we consider a second DGP with an additional stochastic error term ε_{jN},

$$\tilde{z}^*_{jN} = \tilde{z}_{jN} + \varepsilon_{jN} = \alpha_0 + \alpha_w(l_N + w_N) + \alpha_t t_{jN} - M_{jN} + \varepsilon_{jN},$$

with $\varepsilon_{ij} \sim i.i.d.N(0, \sigma^2_\varepsilon)$. We calibrate σ^2_ε such that, in each experiment, the explanatory power as measured by the R^2 is 80% $(= (1 - \sigma^2_\varepsilon/\sigma^2_{\tilde{z}^*}) \times 100\%)$, which is representative of a vast amount of empirical work on gravity models. The term ε_{ij} adds stochastics in a narrow sense which provides for a residual with Models (SILS) and (CF-OLS) and one beyond the approximation (or linearization) error in Model (RF-OLS).

We report on the average bias and root-mean-squared error (RMSE) in percent of the true α across all draws per configuration of $\{N; \alpha\}$ in Tables 3 and 4. Both tables are organized in three by two blocks. Each horizontal block contains estimates for the models (SILS), (CF-OLS) and (RF-OLS) for the cases $N = \{30; 60\}$. Vertically, we have three blocks corresponding to $\alpha = \{-2; -4; -9\}$. For each of the six blocks, we report on the estimated structural parameters $\hat{\alpha}_w$ and $\hat{\alpha}_t$; as well as $\hat{\delta}$ for CF-OLS, the coefficient on the control function.

Table 3 reports the results for the DGP without ε_{ij}; that is, only with the approximation error u_{ij} present. In the absence of ε_{ij}, both models (SILS) and (CF-OLS) correspond to the true one so that both the bias and the RMSE for $\{\hat{\alpha}_w; \hat{\alpha}_w\}$ in percent are zero. That is, the CF-OLS estimator of the linearized gravity model is optimal in the sense of suffering zero efficiency loss due to the linearization. Recall that conditioning on $W_{NN} z_{jN}$ means conditioning on u_{jN}, according to (6). Thus, all the linearization bias is picked up by the coefficient $\hat{\delta}$. Only the RF-OLS approach has an estimation residual and therefore a non-zero variance under this DGP. The RF-OLS estimates of α exhibit some bias, but both bias and RMSE are relatively small. For instance, the largest bias in absolute value over all DGP settings and parameters is only -1.21 percent. Further, as expected, both the bias and the RMSE of $\{\hat{\alpha}_w; \hat{\alpha}_t\}$ in percent decline as the number of countries rises. As the number of countries increases, it has been shown that the need for controlling for general equilibrium effects and nonlinear trade-cost effects as captured by M_{jN} in (SILS) also declines (see Egger and Staub, 2016).

Table 4 depicts results for the DGP with both approximation error and linearization error. The biases for all three estimators are very small, with the maximum bias in absolute value across all parameter configurations and estimators being 1 percent (for RF-OLS in the DGP with $\alpha = -2$ and $N = 30$). The simple CF-OLS estimator achieves RMSEs that are not much larger than those of the more demanding iterative SILS procedure. Even the yet simpler RF-OLS, while exhibiting the highest RMSE of the three estimators, does quite well by this performance measure.

Finally, in Table 5, we present results where all estimators are based on exponentiated equations and a Poisson (pseudo)-likelihood. The estimation quality deteriorates somewhat across all approaches in that case. For SIP and CF-Pois (the Poisson versions of SILS and CF-OLS), the biases continue to be small, with the maximum absolute bias for CF-Pois being about 2 percent. RMSEs are also only moderately higher. With ε_{ij} being homoscedastic normal, OLS estimation in logs is efficient, so these results are expected. The reduced-form approach is not only dependent on ε_{ij} but also on u_{ij}, which is not only non-normal, but also not fully mean-independent of the regressors. The results in the table show that, when exponentiated, the endogeneity is aggravated and RF-Pois suffers from larger absolute biases, ranging up to 9.28 percent. From the perspective of the linearization error, this speaks for using the reduced-form approach in logs (RF-OLS) rather than in exponentiated form (RF-Pois). In contrast, for the control-function approach the properties of the linearization error are of little relevance, and the choice of CF-OLS or CF-Pois can be based on other considerations, such as the properties of the stochastic error.

5 Conclusions

This paper sheds light on the nature of structural linearized gravity models involving an endogenous network-weighted lag – other countries' population-share – of bilateral trade flows as developed in Behrens, Ertur and Koch (2012). We demonstrate that the properties of the network model is such that it can be estimated without any use of network-econometric tools. Exporter-population-share-weighted log bilateral exports on the right-hand side of the models serve as a control function for the approximation error of the linearization, and this variable can be included without specific treatment (i.e., ignoring its endogeneity). These results should please the applied researcher, since estimation of such linearized models only involves OLS (on log-transformed trade flows) with clustered standard errors at the level of exporters.

References

Anderson JE, van Wincoop E. 2003. Gravity with gravitas: a solution to the border puzzle. *American Economic Review* **93**: 170–192.

Arkolakis C, Costinot A, Rodríguez-Clare A. 2012. New trade models, same old gains? *American Economic Review* **102**: 94–130.

Balistreri EJ, Hillberry RH. 2007. Structural estimation and the border puzzle. *Journal of International Economics* **72**: 451–463.

Baltagi BH, Egger PH, Pfaffermayr M. 2015. Panel data gravity models of international trade. In Baltagi BH (ed.) *The Oxford Handbook of Panel Data*. Oxford, UK: Oxford University Press, 608–641.

Behrens K, Ertur C, Koch W. 2012. 'Dual' gravity: Using spatial econometrics to control for multilateral resistance. *Journal of Applied Econometrics* **27**: 773–794.

Bergstrand JH, Egger PH, Larch M. 2013. Gravity redux: estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs. *Journal of International Economics* **89**: 110–121.

Dixit AK, Stiglitz JE. 1977. Monopolistic competition and optimum product diversity. *American Economic Review* **67**: 297–308.

Eaton J, Kortum SS. 2002. Technology, geography, and trade. *Econometrica* **70**: 1741–1779.

Egger PH, Staub KE. 2016. GLM estimation of trade gravity models with fixed effects. *Empirical Economics* **50**: 137–175.

Kelejian HH, Prucha IR. 1998. A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbances. *Journal of Real Estate Finance and Economics* **17**: 99–121.

Kelejian HH, Prucha IR, Yuzefovich Y. 2004. Instrumental variable estimation of a spatial autoregressive model with autoregressive disturbances: large and small sample result. In LeSage J, Pace RK (eds.) *Advances in Econometrics*. New York: Elsevier, 163–198.

Leamer EE, Levinsohn J. 1995. International trade theory: the evidence. In Grossman GM, Rogoff K (eds.) *Handbook of International Economics Vol. 3*. North Holland, 1339–1394.

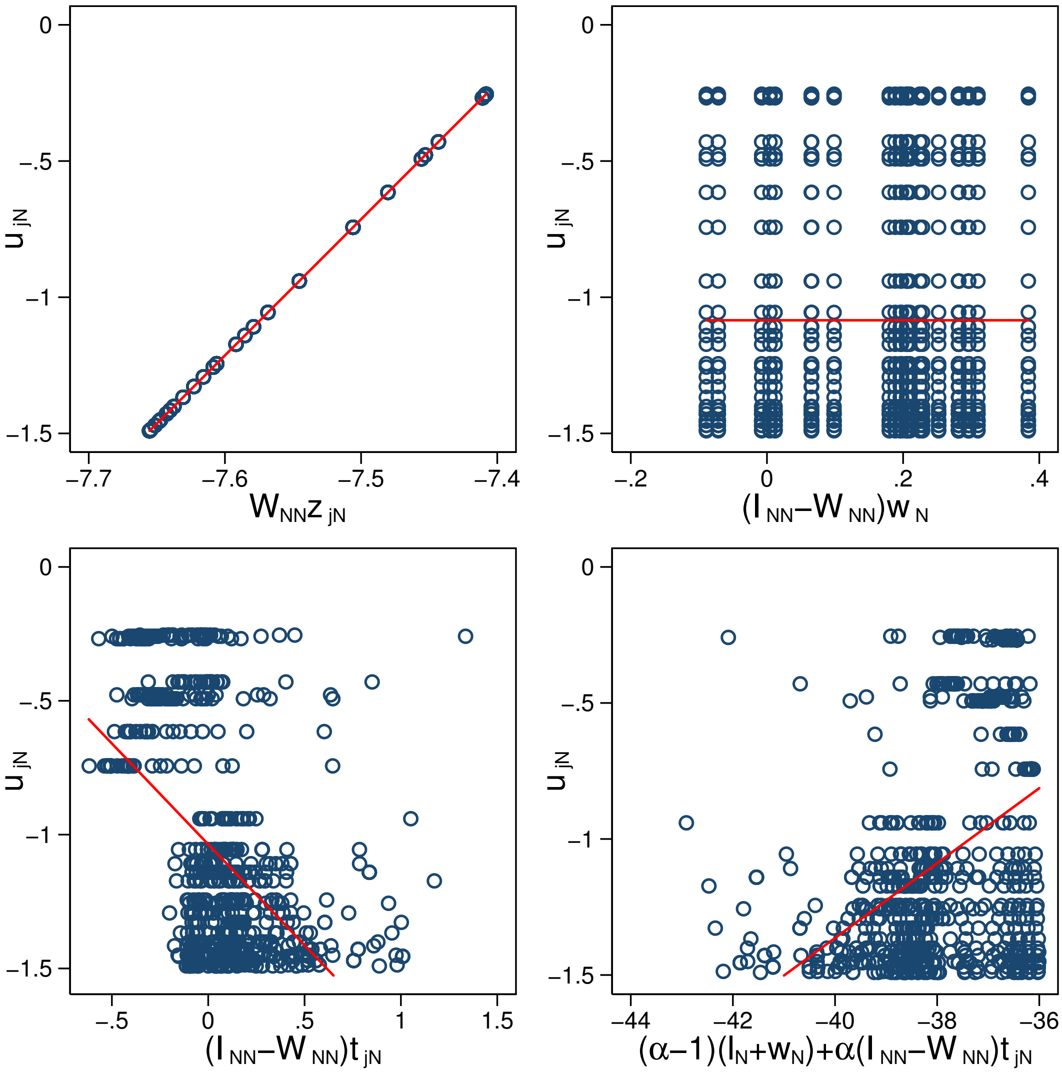
Lee LF. 2003. Best spatial two-stage least squares estimators for a spatial autoregressive model with autoregressive disturbances. *Econometric Reviews* **22**: 307–335.

Santos Silva JMC, Tenreyro S. 2006. The log of gravity. *Review of Economics and Statistics* **88**: 641–658.

Wooldridge JM. 2015. Control function methods in applied econometrics. *Journal of Human Resources* **50**: 420–445.

Tables and figures

Figure 1: Scatterplot and linear fit of approximation error (u) and right-hand side variables from a random draw of the DGP with $\alpha = -4$ for $N = 30$



***Notes*:** The four panels of the figure display scatterplots of data obtained from one random draw of the DGP with $\alpha = -4$ for 30 countries (900 observations). The red line represents the fit from a linear regression.

Table 1: Analysis of variance for key model variables (mean sums of squares over 1,000 replications)

	$N=30$				$N=60$			
SS	z_{ij}	u_{ij}	w_i	$\ddot{t}_{ij}$	z_{ij}	u_{ij}	w_i	$\ddot{t}_{ij}$
				$\alpha=-2$				
i (exporter)	112.26	0.00	14.99	20.47	370.46	0.00	52.21	77.74
j (importer)	112.26	68.40	0.00	29.39	370.46	171.00	0.00	95.89
residual	738.85	0.00	0.00	184.71	2398.63	0.00	0.00	599.66
total	963.38	68.40	14.99	234.58	3139.55	171.00	52.21	773.29
				$\alpha=-4$				
i (exporter)	127.93	0.00	6.01	5.11	416.30	0.00	20.81	19.47
j (importer)	127.93	205.19	0.00	7.47	416.30	519.76	0.00	24.03
residual	737.95	0.00	0.00	46.12	2400.35	0.00	0.00	150.02
total	993.82	205.19	6.01	58.70	3232.95	519.76	20.81	193.52
				$\alpha=-9$				
i (exporter)	139.89	0.00	1.63	1.01	458.94	0.00	5.64	3.84
j (importer)	139.89	949.64	0.00	1.48	458.94	2481.73	0.00	4.78
residual	737.95	0.00	0.00	9.11	2398.50	0.00	0.00	29.61
total	1017.72	949.64	1.63	11.60	3316.38	2481.73	5.64	38.23

Notes: SS refers to sum of squares. $\ddot{t}_{ij} \equiv t_{ij} - \sum_i \frac{L_i}{L} t_{ij}$ is a typical element of $(I_{NN} - W_{NN})t_{jN}$.

Table 2: Partial correlation coefficients of model variables with approximation error u_{jN} (mean and standard deviations over 1,000 replications)

	$N=30$		$N=60$	
	Mean	SD	Mean	SD
		$\alpha=-2$		
$W_{NN}z_{jN}$	1.00	0.00	1.00	0.00
$\tilde{w}_N$	-0.00	0.00	0.00	0.00
$\ddot{t}_{jN}$	0.06	0.14	0.06	0.10
		$\alpha=-4$		
$W_{NN}z_{jN}$	1.00	0.00	1.00	0.00
$\tilde{w}_N$	-0.00	0.00	-0.00	0.00
$\ddot{t}_{jN}$	-0.01	0.19	-0.02	0.14
		$\alpha=-9$		
$W_{NN}z_{jN}$	1.00	0.00	1.00	0.00
$\tilde{w}_N$	0.00	0.00	-0.00	0.00
$\ddot{t}_{jN}$	-0.06	0.21	-0.08	0.17

Table 3: Average bias and root mean squared error of estimated model parameters (1,000 replications): DGP with approximation error only

		$N = 30$			$N = 60$		
		SILS	CF-OLS	RF-OLS	SILS	CF-OLS	RF-OLS
				$\alpha = -2$			
$\hat{\alpha}_w$	Bias	-0.00	0.00	-1.21	-0.00	-0.00	-0.76
	RMSE	0.00	0.00	2.05	0.00	0.00	1.28
$\hat{\alpha}_t$	Bias	-0.00	-0.00	-0.79	-0.00	-0.00	-0.60
	RMSE	0.00	0.00	1.56	0.00	0.00	1.05
$\hat{\delta}$	Bias	–	-150.00	–	–	-150.00	–
	RMSE	–	150.00	–	–	150.00	–
				$\alpha = -4$			
$\hat{\alpha}_w$	Bias	-0.00	0.00	-0.81	-0.00	-0.00	-0.36
	RMSE	0.00	0.00	1.67	0.00	0.00	0.99
$\hat{\alpha}_t$	Bias	-0.00	-0.00	-0.13	-0.00	-0.00	0.08
	RMSE	0.00	0.00	1.80	0.00	0.00	1.24
$\hat{\delta}$	Bias	–	-125.00	–	–	-125.00	–
	RMSE	–	125.00	–	–	125.00	–
				$\alpha = -9$			
$\hat{\alpha}_w$	Bias	-0.00	-0.00	-0.60	-0.00	0.00	-0.16
	RMSE	0.00	0.00	1.46	0.00	0.00	0.91
$\hat{\alpha}_t$	Bias	-0.00	0.00	0.40	-0.00	0.00	0.64
	RMSE	0.00	0.00	2.25	0.00	0.00	1.74
$\hat{\delta}$	Bias	–	-111.11	–	–	-111.11	–
	RMSE	–	111.11	–	–	111.11	–

Notes: Columns SILS, CF-OLS, and RF-OLS refer to structural iterative least squares, control-function OLS and reduced-form OLS estimates of models (SILS), (CF-OLS) and (RF-OLS) in Section 4.3. Table entries are average biases and root mean squared errors in percent of the true α.

Table 4: Average bias and root mean squared error of estimated model parameters (1,000 replications): DGP with approximation error and stochastic error

		$N = 30$			$N = 60$		
		SILS	CF-OLS	RF-OLS	SILS	CF-OLS	RF-OLS
				$\alpha = -2$			
$\hat{\alpha}_w$	Bias	0.20	-0.00	-1.00	-0.03	-0.18	-0.80
	RMSE	4.13	4.11	4.49	1.70	1.70	2.10
$\hat{\alpha}_t$	Bias	0.01	-0.07	-0.79	0.01	-0.12	-0.59
	RMSE	1.00	1.11	1.76	0.43	0.59	1.13
$\hat{\delta}$	Bias	–	-128.50	–	–	-128.95	–
	RMSE	–	129.33	–	–	129.51	–
				$\alpha = -4$			
$\hat{\alpha}_w$	Bias	0.20	0.12	-0.61	-0.05	-0.04	-0.41
	RMSE	3.32	3.30	3.62	1.38	1.38	1.68
$\hat{\alpha}_t$	Bias	0.00	0.33	-0.14	0.00	0.24	0.09
	RMSE	1.04	1.37	1.99	0.45	0.77	1.31
$\hat{\delta}$	Bias	–	-114.20	–	–	-114.57	–
	RMSE	–	114.42	–	–	114.72	–
				$\alpha = -9$			
$\hat{\alpha}_w$	Bias	0.15	0.12	-0.45	-0.05	0.03	-0.20
	RMSE	2.94	2.94	3.22	1.22	1.22	1.51
$\hat{\alpha}_t$	Bias	0.00	0.63	0.39	0.00	0.50	0.65
	RMSE	1.06	1.67	2.41	0.46	1.01	1.80
$\hat{\delta}$	Bias	–	-106.46	–	–	-106.70	–
	RMSE	–	106.51	–	–	106.73	–

Notes: Columns SILS, CF-OLS, and RF-OLS refer to structural iterative least squares, control-function OLS and reduced-form OLS estimates of models (SILS), (CF-OLS) and (RF-OLS) in Section 4.3. Table entries are average biases and root mean squared errors in percent of the true α.

Table 5: Poisson PML estimators — Average bias and root mean squared error of estimated model parameters (1,000 replications): DGP with approximation error and stochastic error

		$N = 30$			$N = 60$		
		SIP	CF-Pois	RF-Pois	SIP	CF-Pois	RF-Pois
				$\alpha = -2$			
$\hat{\alpha}_w$	Bias	0.31	-2.15	-9.28	0.02	-1.94	-7.08
	RMSE	6.69	5.70	13.62	2.65	3.00	9.18
$\hat{\alpha}_t$	Bias	0.07	-1.46	-6.00	0.03	-1.32	-4.49
	RMSE	3.36	2.78	7.90	1.13	1.82	5.27
$\hat{\delta}$	Bias	–	-129.90	–	–	-130.49	–
	RMSE	–	130.60	–	–	130.95	–
				$\alpha = -4$			
$\hat{\alpha}_w$	Bias	0.28	-1.78	-7.92	-0.01	-1.60	-5.99
	RMSE	5.65	4.61	11.61	2.21	2.46	7.91
$\hat{\alpha}_t$	Bias	0.09	-0.37	-4.05	0.02	-0.46	-2.75
	RMSE	3.83	2.86	7.59	1.24	1.49	4.55
$\hat{\delta}$	Bias	–	-115.05	–	–	-115.28	–
	RMSE	–	115.23	–	–	115.41	–
				$\alpha = -9$			
$\hat{\alpha}_w$	Bias	0.24	-1.64	-7.41	-0.02	-1.40	-5.56
	RMSE	5.28	4.12	10.66	1.97	2.17	7.32
$\hat{\alpha}_t$	Bias	0.11	0.39	-2.58	0.02	0.19	-1.36
	RMSE	4.44	3.48	8.08	1.34	1.73	4.80
$\hat{\delta}$	Bias	–	-106.92	–	–	-107.02	–
	RMSE	–	106.95	–	–	107.05	–

Notes: Columns SIP, CF-Pois, and RF-Pois refer to structural iterative Poisson PML, control-function Poisson PML and reduced-form Poisson PML estimates of (exponentiated versions of) models (SILS), (CF-OLS) and (RF-OLS) in Section 4.3. Table entries are average biases and root mean squared errors in percent of the true α.